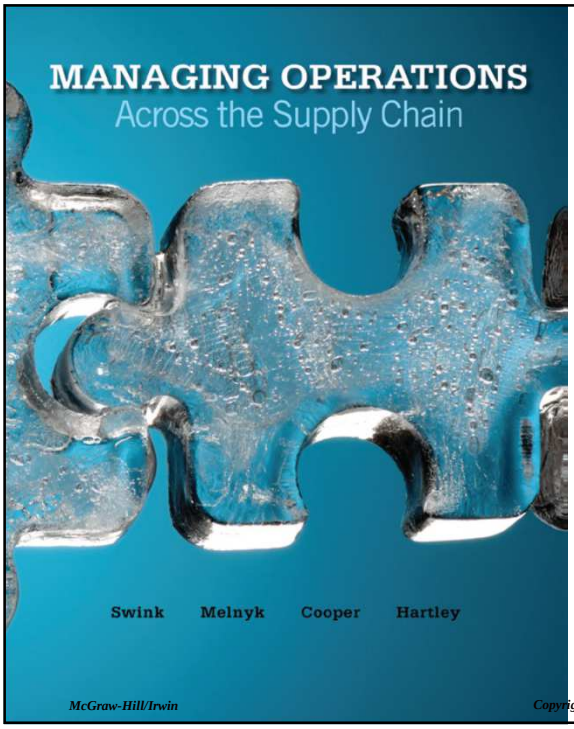

Technical Supplement 2

Waiting Line Models

Learning Objectives

1. Explain the importance of waiting line models to service operations management.
2. Give a real example of a decision modeled with waiting line methods.
3. Demonstrate the measures important in waiting line situations.
4. Show how waiting line models can be solved.

New York City Justice System

- Severe backlog
- Need to reduce average wait to 24 hours
- Analysis
  - Identification of system
  - Data gathering
  - Modeling – display estimated performance

Used Monte Carlo simulation for waiting line problem due to complexity

TS2-3

Example Waiting Line Systems

	Arrivals	Queue	Service
Barber shop	Customers	Assigned numbers	Barbers
Car wash	Cars	Line	Washers
Computer center	Jobs	Stacked jobs	CPU
Shipping dock	Ships	Empty or load	Cranes
Telephone system	Calls	Lines	Switchboard

TS2-4

Waiting Line Systems

- COMPONENTS
  - –Arrivals
    - Calling population ∞ limit K
    - Distribution
  - –Discipline FIFO LIFO priority
  - –Services
    - Number of servers
    - Distribution
  - –Departures

TS2-5

Waiting Line Decisions

- Steady State:
  - –performance at equilibrium
- Transient State:
  - –temporary characteristics on way to equilibrium
- Total Cost
  - –balance
    - Cost of providing service
    - Cost of waiting

TS2-6

Source of Waiting Line System Performance Differences

- Variance
  - In arrival pattern
  - In service times
- If constant system, calculations simple
- If variance, system will have idle periods balanced with periods where work is waiting
- Need to overbuild if variance exists

TS2-7

Impact of Busy System

- If rate of arrival ≥ rate of service capacity
  - System is degenerate (cannot get work done)
- If rate of arrival is close to service capacity
  - System will experience long waiting lines at some times should variance exist

TS2-8

System Measures

- P_n probability of n in the system
- W_q average time waiting for service
- L_q average number waiting for service
- W average time in system (including service)
- L average number in system (including being served)
- ρ system utilization rate

TS2-9

Waiting Line Model Assumptions

- Arrival distribution
- Service distribution
- Number of servers
- Queue discipline
- Maximum number of customers in system
- Number of potential customers

TS2-10

Arrival Distributions

- M Exponential (or Poisson)
- D Deterministic (constant)
- E_k Erlang (shape parameter k)
- GI General independent
- Others

λ = number of arrivals per unit time

TS2-11

Service Distributions

- M Exponential (or Poisson)
- D Deterministic (constant)
- E_k Erlang (shape parameter k)
- GS General service
- Others

μ = Mean service time

TS2-12

Other Assumptions

- S number of servers (clones)
- Queue discipline
  - –FIFO (FCFS) first in, first out
  - –LIFO last in, first out
  - –SIRO random
  - –GD general distribution (other rules)
- Max Line Length
- Calling Population

TS2-13

Standard Models

- M/M/1:FCFS/∞ /∞
- M/D/1:FCFS/∞ /∞ constant service
- M/GS/1:FCFS/∞ /∞ normal service
- M/M/s:FCFS/∞ /∞ multiple servers
- M/M/1:FCFS/k/∞ max k in system
- M/M/s:FCFS/k/∞/ multiple servers, max k
- others

TS2-14

Waiting Line Models Summary

1. Many service operations involve waiting lines.
2. Waiting line models good to understand counterintuitive behavior of such systems.
3. For complex systems, use simulation.

TS2-15