


## Learning Objectives

1. Describe elements of an inventory policy
2. Explain differences between inventory planning models
3. Calculate inventory policy parameters
4. Determine cost of firm's service level
5. Explain cost/benefit of strategies
6. Describe inventory planning techniques

## Inventory Control Objectives

- We need to answer the following questions in order to balance supply and demand, and balance costs and service levels.
-When do I order?
-How much do I order?
-Where do I deploy the inventory?




## Continuous Review Model

- Continuous Review: inventory levels are constantly monitored to determine when to place a replenishment order



## Total Acquisition Costs

- Total Acquisition Costs: sum of all relevant annual inventory costs
- Holding costs: associated with storing and assuming risk of having inventory
- Ordering costs: associated with placing orders and receiving supply

TAC = annual ordering cost + annual carrying cost

## Total Acquisition Costs

TAC = annual ordering cost + annual carrying cost

$$
=\mathrm{C}_{0}(\mathrm{D} / \mathrm{Q})+\mathrm{UC}_{i}{ }^{*} \mathrm{Q} / 2
$$

$$
N=D / Q
$$

$$
\mathrm{I}=\mathrm{Q} / 2
$$

## Where:

$\mathrm{N}=$ orders per year
I = average inventory level
$\mathrm{D}=$ annual demand
$\mathrm{C}_{0}=$ order cost per order
$Q=$ order quantity
$\mathrm{U}=$ unit cost
$\mathrm{C}_{i}=\%$ carrying cost per year

## Total Acquisition Costs

If we need 3,000 units per year at a unit price of \$20 and we order 500 each time, at a cost of $\$ 500$ per order with a carrying cost of $20 \%$, what is the TAC?

$$
\begin{aligned}
& N=D / Q=3000 / 500=6 \text { order per year } \\
& I=Q / 2=500 / 2=250 \text { average inventory }
\end{aligned}
$$

TAC $=$ ordering cost + carrying cost
$=\mathrm{C}_{o}(\mathrm{D} / \mathrm{Q})+\left(\mathrm{U} \mathrm{C}_{i}\right)(\mathrm{Q} / 2)$
$=\$ 50(3000 / 500)+(\$ 20 * 25 \%)^{*}(500 / 2)=\$ 1,300$
Where:

$$
\begin{array}{lrcr}
\mathrm{N}=\mathrm{D} / \mathrm{Q} & \mathrm{Q}=50 \overline{0} & \mathrm{I}=\mathrm{Q} / 2 & \mathrm{U}=\$ 20 \\
\mathrm{D}=3,000 & \mathrm{C}_{o}=\$ 50 & \mathrm{C}_{i}=25 \% &
\end{array}
$$

## Total Acquisition Costs

If we need 3,000 units per year at a units price of $\$ 20$ and we order $\mathbf{2 0 0}$ each time, at a cost of $\$ 50$ per order with a carrying cost of $20 \%$, what is the TAC?

$$
\begin{aligned}
& N=D / Q=3000 / 200=15 \text { order per year } \\
& I=Q / 2=200 / 2=100 \text { average inventory }
\end{aligned}
$$

TAC $=$ ordering cost + carrying cost

$$
\begin{aligned}
& =\mathrm{C}_{o}(\mathrm{D} / \mathrm{Q})+\left(\mathrm{U} \mathrm{C}_{i}\right)(\mathrm{Q} / 2) \\
& =50(3000 / 200)+\left(\$ 20^{*} 25 \%\right)^{*}(200 / 2)=\$ 1,150
\end{aligned}
$$

Where:
$\mathrm{N}=\mathrm{D} / \mathrm{Q}$
$Q=50 \overline{0}$
$\mathrm{I}=\mathrm{Q} / 2$
$\mathrm{U}=\$ 20$
$D=3,000$
$\mathrm{C}_{0}=\$ 50$
$C_{i}=25 \%$

## Economic Order Quantity (EOQ)

- Economic Order Quantity (EOQ): minimizes total acquisition costs; point at which holding and orders costs are equal


## - How much to order

D = Annual Demand
$\mathrm{C}_{o}=$ Ordering cost
U = Unit cost
$\mathrm{C}_{i}=$ Holding cost

$$
E O Q=\sqrt{\frac{2 D C_{o}}{U C_{i}}}
$$



## Economic Order Quantity (EOQ)

- If we need 3,000 units per year at a unit price of $\$ 20$, at a cost of $\$ 50$ per order with a carrying cost of $20 \%$, what is lowest TAC order quantity?

$$
\begin{aligned}
E O Q & =\sqrt{\frac{2 D C_{o}}{U C_{i}}}
\end{aligned}=\sqrt{\frac{2^{*} 3000^{*} 50}{20^{*} 25 \%}}=273.86 \Rightarrow 274 ~\left(\begin{array}{l}
\mathrm{D}=3,000 \\
\mathrm{C}_{o}=\$ 50 \\
\mathrm{U}^{\mathrm{E}}=\$ 20 \\
\mathrm{C}_{i}=25 \%
\end{array}\right.
$$

## Reorder Point - No Uncertainty

- Reorder Point: minimum level of on-hand inventory that triggers a replenishment
- When to order

$$
R O P=(\bar{d}) \bar{t}
$$

$\overline{\bar{d}}=$ average demand per time period
$t=$ average supply lead time

## Reorder Point - No Uncertainty

If you use 10 units per day, and the lead time for resupply is 9 days, how low can your inventory get before placing a new order?

$$
\begin{aligned}
R O P & =(\bar{d}) \bar{t} \\
& =9^{*} 10 \\
& =90
\end{aligned}
$$

## EOQ Extensions

- Assumptions underlying EOQ:
- No quantity discounts
- No lot size restrictions
- No partial deliveries
- No variability
- No product interactions


## Total Acquisition Costs

$T A C+C_{o}\left(\frac{D}{Q}\right)+U C_{i}\left(\frac{Q}{2}\right)+p D$
$\mathrm{C}_{0}=$ Ordering cost
D = Annual demand
Q = Order quantity
$U=$ Unit cost
$\mathrm{C}_{i}=$ Holding cost
p = unit purchase cost

## Total Acquisition Costs

If we need 3,000 units per year at a unit price of $\$ 19$, at a cost of $\$ 50$ per order with a carrying cost of $20 \%$, what is TAC with a $\mathrm{Q}=1,000$ ?

$$
\begin{aligned}
T A C & =C_{o}\left(\frac{D}{Q}\right)+U C_{i}\left(\frac{Q}{2}\right)+p D \\
& =\$ 50\left(\frac{3,000}{1,000}\right)+\$ 19 * 20 \%\left(\frac{1,000}{2}\right)+\$ 19 * 3000 \\
& =\$ 59,050
\end{aligned}
$$

TAC at unit cost $\$ 20=\$ \mathbf{6 1 , 0 9 8}$. new price saves $\mathbf{\$ 2 , 0 4 8}$
Where: $\mathrm{C}_{o}=\$ 50$
D $=3,000$
$\mathrm{Q}=1,000$
$\mathrm{U}=\$ 19$
C $i=20 \%$
$\mathrm{p}=\$ 19$

## Price Discounts \& Lot Sizes

- Determining best price break quantity:
1.Identify price breaks/lot size restrictions
2.Calculate EOQ for each price/lot size
3.Evaluate viability of each option
4.Calculate TAQ for each option
5.Select best TAQ option


## Production Order Quantity

- Production Order Quantity: most
economical order quantity when units become available at rate produced

D = Annual Demand
$\mathrm{C}_{o}=$ Ordering cost
$\mathrm{C}_{i}=$ Holding cost
U = Unit cost
d = daily rate of demand
$p=$ daily rate of production

$$
Q_{p}=\sqrt{\frac{2 D C_{o}}{C_{i} U\left(1-\frac{d}{p}\right)}}
$$

## Production Order Quantity

$\mathrm{Q}_{p}=\mathrm{EOQ}$

$$
U=\$ 10
$$

$$
d=2,000
$$

$$
p=5,000
$$

$$
\begin{aligned}
Q_{p} & =\sqrt{\frac{2 D C_{o}}{C_{i} U\left(1-\frac{d}{p}\right)}} \\
& =\sqrt{\frac{2 * 500,000 * \$ 2,000}{25 \% * \$ 10\left(1-\frac{2,000}{5,000}\right)}} \\
& =36,514.84=36,515
\end{aligned}
$$

## Demand During Lead Time

- Variation can occur in both demand rates and lead times

$$
\sigma_{d d l t}=\sqrt{\bar{t} \sigma_{d}^{2}+\bar{d}^{2} \sigma_{t}^{2}}
$$

$\sigma_{\text {ddlt }}=$ standard deviation of demand during lead time
$\bar{t}=$ average lead time
$\sigma_{d=}^{2}$ standard deviation of demand
$\bar{d}=$ average demand
$\sigma_{t}^{2}=$ standard deviation of lead time

## Demand During Lead Time

Average demand is 10 units day with standard deviation of 1.5 , and lead time of 10 days with standard deviation of 2.5 days

$$
\begin{aligned}
& \sigma_{d d l t} & =\sqrt{\bar{t} \sigma_{d}^{2}+\bar{d}^{2} \sigma_{t}^{2}} \\
\bar{t}=10 \text { days } & & =\sqrt{9\left(1.5^{2}\right)+10^{2}\left(2.5^{2}\right)} \\
\sigma_{d}^{2}=1.5 \text { units } & & =25.4 \text { units } \\
\bar{d}=10 \text { per day } & & \\
\sigma_{t}^{2}=2.5 \text { days } & &
\end{aligned}
$$

## Determining Service Levels

- Service Level Policy: determining the acceptable stockout risk level

$$
S S=z \sigma_{d d l t}
$$

SS = Safety stock
$z=$ standard deviations needed for service level
$\sigma_{d d l t}=$ standard deviation of demand during lead time

## Determining Service Levels

Standard deviation of demand during lead time is 25.4 units, acceptable stock out level is $5 \%$ ( $95 \%$ service level). From the $z$ table $=1.65$

$$
\begin{aligned}
S S & =z \sigma_{\text {ddlt }} \\
& =1.65 * 25.4 \\
& =42 \text { units }
\end{aligned}
$$

Safety stock carrying cost:
$\$ 19 * 42$ units $* 20 \%=\$ 159.60$ year

## Economic Order Quantity (EOQ)

FIGURE 14-4
Relationship between
Inventory Investment
and Product Availability


## Revisiting ROP and Average Inventory

## - Considering uncertainty

ROP $=$ Reorder point
$\bar{d}=$ average lead time

$$
R O P=(\bar{d} x \bar{t})+S S
$$

$\bar{t}=$ average demand
SS = Safety stock
Q = order quantity

$$
\text { average inventry }=\frac{Q}{2}+S S
$$

$$
R O P=(10 * 9)+42=132 \text { units }
$$

$$
\text { average inventory }=\left(\frac{1000}{2}\right)+42=542
$$

## Periodic Review Period

- Order Interval: fixed time between inventory review, on-hand level is unknown during this uncertainty period

UP = Uncertainty period
$U P=O I+\bar{t}$
OI = Order interval
$\bar{t}=$ average lead time
$Q=\bar{d}(U P)+z \sigma_{d d l t} \sqrt{U P}-A$
$\bar{d}=$ average lead time
$z=$ standard deviations needed for service level
$\sigma_{\text {ddlt }}=$ standard deviation of demand during lead time
A = inventory on hand

## Periodic Review Model

- Orders are placed every 30 days and average lead time is 9 days. Standard deviation of demand is 1.5 units.

$$
\begin{array}{lrl}
\mathrm{UP}=\text { Uncertainty period } & U P & =O I+\bar{t} \\
\begin{array}{cl}
O I & =30 \text { days } \\
\bar{t} & =9 \text { days } \\
\sigma_{d} & =1.5 \text { units }
\end{array} & & =30+9=39 \text { days } \\
& & \sigma_{d d l t}
\end{array}=\sqrt{(U P) \sigma_{d}^{2}} .
$$

## Periodic Review Period

- There are currently 105 units in stock

$$
\begin{aligned}
\mathrm{UP} & =39 \text { days } & Q & =\bar{d}(U P)+z \sigma_{d d l t} \sqrt{U P}-A \\
O I & =30 \text { days } & & \\
\bar{t} & =9 \text { days } & & =10(39)+1.65(9.37) \sqrt{39}-105 \\
\bar{d} & =10 \text { units } & & =95 \%=1.65 \\
\sigma_{\text {ddlt }} & =9.37 & & =390+97-105 \\
\mathrm{~A} & =105 & & =382 \text { units }
\end{aligned}
$$

## Single Period Inventory Model

- Single Period Inventory Model: items are ordered once, and have little left over value (newsvendor problem)
- Target Service Level: probability of meeting demand

$$
\begin{gathered}
C_{\text {stockout }}=\text { Unit selling price }- \text { Unit } \cos t \\
C_{\text {overstock }}=\text { Unit } \cos t+\text { Disposal } \cos t-\text { Salvage } \cos t \\
(1-T S L)\left(C_{\text {stockout }}\right)=T S L\left(C_{\text {overstock }}\right) \\
T S L=\frac{C_{\text {stockout }}}{C_{\text {stockout }}+C_{\text {overstock }}}
\end{gathered}
$$

## Single Period Inventory Model

- Units cost $\$ 10$ and sell for $\$ 30$, unsold units have no value, and no disposal or salvage value

$$
C_{\text {stockout }}=\text { Unit selling price }- \text { Unit } \cos t
$$

$C_{\text {overstock }}=$ Unit $\cos t+$ Disposal $\cos t-$ Salvage $\cos t$

$$
\begin{aligned}
T S L & =\frac{C_{\text {stockout }}}{C_{\text {stockout }}+C_{\text {overstock }}} \\
C_{\text {so }} & =\$ 30-\$ 10=\$ 20 \\
C_{o s} & =\$ 10-0-0=\$ 10 \\
T S L & =\frac{\$ 20}{\$ 20+\$ 10}=.667
\end{aligned}
$$

## Impact of Location on Inventory

- Square Root Rule: estimation of impact of changing the number of locations on inventory

$$
S S_{n}=\frac{\sqrt{N_{n}}}{\sqrt{N_{e}}} \times S S_{e}
$$

$S S_{n}=$ safety stock of the new number of locations
$N_{n}=$ total number of new locations
$N_{e}=$ number of existing locations
$S S_{e}=$ system safety stock for existing locations

## Impact of Location on Inventory

- A single warehouse currently has 1,000 units of safety stock. How much is needed if a second warehouse is added?
$S S_{n}=$ safety stock of the new number of locations

$$
N_{n}=2
$$

$N_{e}=1$
$S S_{e}=1000$

$$
\begin{aligned}
S S_{n} & =\frac{\sqrt{N_{n}}}{\sqrt{N_{e}}} \times S S_{e} \\
& =\frac{\sqrt{2}}{\sqrt{1}} \times 1000 \\
& =1,410
\end{aligned}
$$



## Reducing Inventory Costs

- Managing Cycle Stock: reducing lot sizes
- Managing Safety Stock: using ABC analysis and reducing lead time
- Managing Locations: balance inventory, lead time and service levels
- Implementing Inventory Models: matching management system to specific items


## Independent Demand Inventory Planning

1. Determines how much and when to order
2. Continuous systems monitor on-hand levels
3. Safety stock levels are linked with service levels
4. Periodic systems count inventory at specific intervals
5. Inventory policy parameters vary by model
6. Production and economic order quantities are similar
7. Number of storage locations impact inventory levels
8. Managers should work to reduce inventory levels
