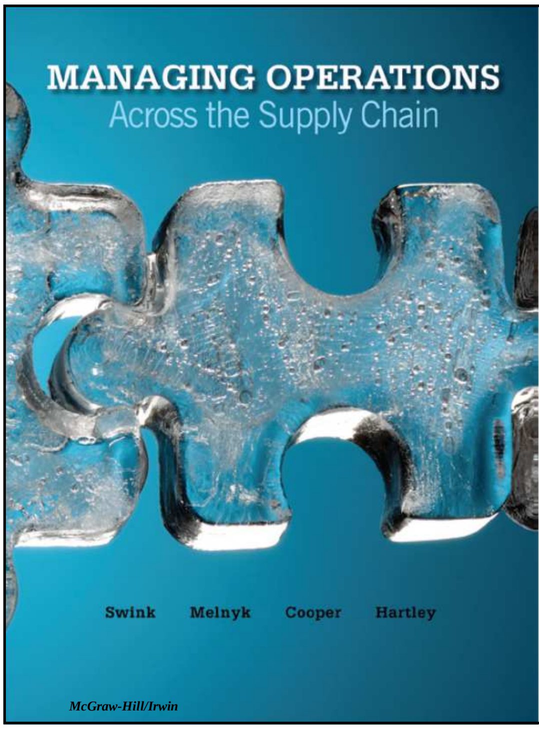


CHAPTER TWELVE SUPPLEMENT

Advanced Forecasting and Tracking Models

Learning Objectives

1. Develop forecasts using enhanced methods
2. Evaluate and select forecast models using error metrics
3. Explain how the parameters of forecasting models respond to changed in demand

Exponential Smoothing with Trend Effects

- More responsive to changes than exponential smoothing

$$FIT_t = F_{t+1} + T_{t+1}$$

$$F_{t+1} = FIT_t + \alpha(d_t - FIT_t)$$

$$T_{t+1} = T_t + \delta(F_{t+1} - FIT_t)$$

Where:
FIT_t = forecast including trend for period t
F_t = "base" forecast for period t from simple model
T_t = forecast trend component of demand for period t
α = base smoothing constant
δ = trend smoothing constant

12S–3

Exponential Smoothing with Trend Effects

$$F_{t+1} = 250 + 0.20(270 - 250) = 254$$

$$T_{t+1} = 10 + 0.10(254 - 250) = 10.4$$

$$FIT_t = 254 + 10.4 = 264.4$$

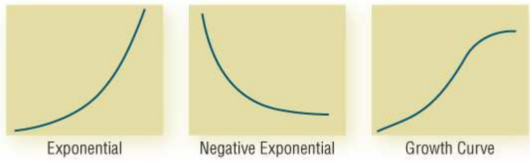
Where:
FIT_t = 250 units
F_t = 10
T_t = 270
α = 0.20
δ = 0.10

Example 12S-1

12S–4

Determining Trend Factors

FIGURE 12S-1
Common Nonlinear Trends



$$d_t = a + b * t$$

Where:

- d_t = demand value for period t
- t = number of periods form origin
- a = y-axis intercept of line
- b = slope of line

Figure 12S-1

Simple Linear Regression: Time Series

$$b = \frac{\sum_{t=1}^{t=n} t d_t - n\bar{t} * \bar{d}_t}{\sum_{t=1}^{t=n} t^2 - n\bar{t}^2} \qquad \text{(12S-6)}$$

$$a = \bar{d}_t - b\bar{t} \qquad \text{(12S-7)}$$

Where:

- d_t = actual demand value for period t
- F_t = forecast demand from regression equation in period t
- $\bar{t}$ = average of all t values
- $\bar{d}_t$ = average of all d_t values
- n = number of data points

Simple Linear Regression: Time Series

Using the data set from Table 12S-1

$$b = \frac{[17{,}782.1 - (16)(8.5)(127)]}{[1{,}496 - (16)(8.5)^2]} = 1.5 \; per \; period$$

$$a = (127) - (1.51)(8.5) = 114.2$$

$$F_{17} = a + bt = 114.2 + 1.5 * 17 = 139.8$$

Where:

$d_t = 16$

$t*d = 17{,}785$

$t = 8.5$

$d_t = 127$

$n = 16$

Example 12S-2

12S–7

Seasonality

Seasonal Index: adjustment factor to account for seasonal changes or cycles in demand. A ‘season’ can be any time period.

SI = period actual demand / average period demand

Average SI = Average of SI data for given time frame

12S–8

Seasonality

1. Gather actual demand data
2. Calculate average period demand
3. Calculate seasonal index (actual/avg)
4. Calculate average seasonal index

Week	Day	Actual Demand (a)	Avg Wkly Demand (b)	SI (a/b)	Avg SI
Week 1	Mon	123.6	159.1	0.78	
	Tue	134.9		0.85	
	Wed	160.0		1.01	
	:	:		:	
Week 2	Mon	145.9	167.8	0.87	
	Tue	130.0		0.77	
	Wed	145.0		0.86	
	:	:		:	
Week 3	Mon	159.0	197.8	0.80	0.82
	Tue	178.7		0.90	0.84
	Wed	160.0		0.81	0.89
	:	:		:	:

5. Determine forecast
6. Apply average SI
7. Calculate seasonally adjusted forecast

Week	Day	Base Forecast (a)	Avg SI (b)	Adjusted Forecast (a*b)
Week 4	Mon	218.8	0.89	194.7
	Tue	222.8	0.90	200.5
	Wed	226.9	0.93	211.02
	:	:	:	:

12S–9

Forecast Error and Signal Tracking

Error Measure	Definition	Formula
Bias = Mean Forecast Error (MFE)	The average of the deviations of observed values from the forecasted values	$MFE = \frac{\sum_{t=1}^{n}(d_t - F_t)}{n}$
Mean Percent Error (MPE)	The average forecast error restated as a percentage deviation	$MPE = \frac{\sum_{t=1}^{n}\frac{(d_t - F_t)}{d_t}*100}{n}$
Mean Absolute Deviation (MAD)	The average of the absolute values of the deviations of the observed values from the forecasted values	$MAD = \frac{\sum_{t=1}^{n}\lvert d_t - F_t\rvert}{n}$
Mean Absolute Percentage Error (MAPE)	The MAD adjusted to create a relative metrics that indicate how large errors are relative to the actual demand quantities	$MAPE = \frac{\sum_{t=1}^{n}\frac{\lvert d_t - F_t\rvert}{d_t}*100}{n}$
Mean Squared Error (MSE)	A measure of forecast error that enables the user to evaluate the sensitivity of a forecast to the magnitude of the errors	$MSE = \frac{\sum_{t=1}^{n}(d_t - F_t)^2}{n-1}$
Root Mean Squared Error (RMSE)	The square root of the MSE, a measure that usually give an approximation of the variance of errors	$RMSE = \sqrt{MSE}$

Where d_t = Actual Demand at Time t and F_t = Forecast at Time t

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Forecast Error and Signal Tracking

TABLE 12S-6 Assessing Forecast Accuracy: A Comparison of MAD, RMSE, and Standard Deviation

Period	Actual	Forecast	Forecast Error	|Actual − Forecast|	Error Square
1	345	340	5	5	25
2	328	341	−13	13	156
3	335	339	−4	4	18
4	330	339	−9	9	78
5	334	338	−4	4	16
6	340	338	2	2	6
7	338	338	0	0	0
8	328	338	−10	10	96
9	345	337	8	8	67
10	350	338	12	12	153
			8.2	6.7	8.3
			STD DEV	MAD	RMSE

Table 12S-6

12S–11

Forecast Error and Signal Tracking

Tracking Signal: ratio of running forecast error to MAD

$$\text{Tracking signal} = \frac{\sum_{t=1}^{n}(d_t - F_t)}{MAD_{t=1\rightarrow n}}$$

FIGURE 12S-4 Tracking Signal Control Chart

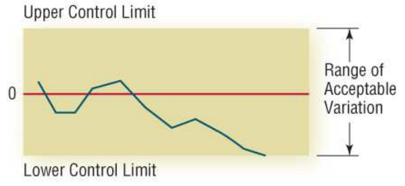

Figure 12S-4

12S–12

Advanced Forecasting Models Summary

1. Enhanced forecasting models can better respond to changing demand
2. Multiple measure for determining forecast error
3. Tracking signals can provide notification when forecast error is changing

12S–13